

## Professor Ion Valuță – teacher of many generations of students

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Professor doctor Ion Valuță (Valutse) was born on 29th of May 1930 in Dondușeni (Dondusheni), Soroca district, Romania, today Republic of Moldova. In his "first seven years of life", he acquired not only the main norm of correct social behavior, but also he was conquered by the beauty of Romanian language and by the charm of numbers, he understood the power and the value of the word and developed the sense of responsibility. Studied at the local school in Dondușeni and two years at the Lyceum from Iasi, Romania. His university studies were at the Pedagogical University of Chisinau in 1947-1951 and then in 1954-1955, at the M. V. Lomonosov Moscow University. The doctoral studies were carried under the direction of the great Professor A. G. Kurosh at Mechanical-Mathematical faculty of M. V. Lomonosov Moscow University. In 1963 he received his doctorate, then in 1965 he holds the scientific title of docent, and in 1983 the Higher Attestation Commission of the Soviet Union confirms the title of the Professor. Many years he was in the position of the dean of the Physical-Mathematical faculty of Tiraspol University, where he works in the period of 1952 - 1964. Professor Ion Valuță soon became well known in the Universitario sphere of the Republic of Moldova. The process of industrialization of the republic began and for this purpose was taken the decision to found the Polytechnic University in Chisinau. In 1964 the Academician Serghei Radautsan, Professor Ion Valuță together with others started to create this important university. From 1965 to 1975 he was vice-rector for scientific researcher. During these years, the managerial and professional qualities were fully manifested. An institute with great authority was created.

In the multidimensional activity during the life of Professor Doctor Ion Valuță, the priority directions are the following:

The theory of universal algebras and applications.

History and Methodology of Science.

Didactics and Education.

Management of Scientific Research.

The book [14] contains extensive information about the activity of Professor Ion Valuță. We will expose some moments from the life and activity of Professor Ion Valuță.

**The theory of universal algebras and applications:** Let  $\mathbb{N} = \{1, 2, \dots\}$  be the set of natural numbers and  $\omega = \{0, 1, 2, \dots\}$  be the set of non-negative integers. Let  $n \in \omega$ .

The  $n$ -ary Cartesian power of a set  $X$  is denoted by  $X^n$ . If the set  $X$  is empty, then the set  $X^n$  is empty too. If the set  $X$  is non-empty, then the set  $X^0$  is a singleton and the set  $X^n$  is non-empty too.

The discrete sum  $\Omega = \oplus\{\Omega_n : n \in \omega\}$  of the pairwise disjoint sets  $\{\Omega_n : n \in \omega\}$  is called a signature. A topological  $\Omega$ -algebra or a topological universal algebra of the signature  $\Omega$  is a family  $\{G, e_{nG} : n \in \omega\}$ , where  $G$  is a non-empty set and  $e_{nG} : G^n \rightarrow G$  is a mapping for each  $n \in \omega$ . The concept of universal algebra was created by Alfred North Whitehead in 1898 as a generalization of Boole's logical algebras. The term universal algebra was proposed by James Joseph Sylvester [33]. Between 1935 and 1950 important works were published by Garrett Birkhoff, in which he introduced the notions of variety, quasi-variety, free algebra, congruences and proved the homomorphism theorems [2, 3, 4]. After 1950, due to applications in mathematical logic, model theory, geometric algebras, theoretical and computer physics, the theory of universal algebras began to develop fruitfully [4, 6, 16, 17, 19, 27, 28]. In [22, 23, 25, 26, 12, 27, 12, 10, 11] were studied semigroups of endomorphisms of universal topological algebras.

Let  $A, B$  and  $C$  be three universal algebras of signature  $\Omega$ . The function  $f : A \rightarrow B$  is called a morphism or homomorphism, if  $f(u(x)) = u(f^n(x))$  for any  $x = (x_1, x_2, \dots, x_n) \in G^n$  and  $n \in \omega$ , where  $f^n(x) = (f(x_1), f(x_2), \dots, f(x_n))$  and  $u \in \Omega_n$ . The composition of the functions  $f : A \rightarrow B$  and  $g : B \rightarrow C$  is the function  $h = f \cdot g : A \rightarrow C$ , where  $h(x) = g(f(x))$  for any  $x \in A$ . A morphism that is a bijective function is called an isomorphism. If a isomorphism can be established between two universal algebras, they are called isomorph. Two isomorph universal algebras are identified. Morphisms, respectively isomorphisms, of a topological universal algebra in itself are called endomorphisms, respectively automorphisms. Subalgebras and Cartesian products of  $\Omega$ -algebras are defined in traditional way [4, 6, 16, 27, 28, 33].

The family of all endomorphisms  $End(G)$  of a universal algebra  $G$  relatively to the operation of composition  $f \cdot g$  is a monoid.

Let  $\Omega$  be a fixed signature. For any non-empty subset  $A$  of the universal algebra  $G$  denote by  $s_G(A)$  the smallest subalgebra of  $G$  which contains the set  $A$ . Denote by  $Sub(G)$  the lattice of all subalgebras of  $G$ . We consider that the empty subset is the minimal subalgebra of the algebra  $G$ . A universal algebra  $G$  is called cyclic if there exists a point  $a \in G$  such that the set  $\{a\}$  generate the algebra  $G$ . A universal algebra  $G$  is a free universal algebra in some class (see [15, 16, 6]) of universal algebras if there is given a subspace  $I = I_G \subset G$  with the properties:

1) the algebra  $G$  is generate by the set  $I$ , i.e.  $G = s_G(I)$  and  $I$  is called the space of generators of  $G$ ;

2) for any continuous mapping  $f : I \longrightarrow G$  there exists a (unique) continuous endomorphism  $\tilde{f} : G \longrightarrow G$  such that  $f(x) = \tilde{f}(x)$  for each  $x \in I$ .

Denote by  $Exp(X)$  the lattice of all subsets of  $X$ . Let  $S$  be a multiplication semigroup. A non-empty subset  $A$  of  $S$  is called a left (respectively right) ideal of  $S$  if  $S \cdot A \subset A$  (respectively  $A \cdot S \subset A$ ).

**Lemma 1.** *Let  $G$  be a universal algebra of the signature  $\Omega$  and  $M$  be a subset of  $G$ . Then the set  $End(G)_M = \{\varphi \in End(G) : \varphi(G) \subset M\}$  is a left ideal of the semigroup  $End(G)$ .*

The left ideal  $End(G)_M$  is called a  $G$ -saturated ideal of the monoid  $End(G)$ . Denote by  $Spec^s(S)$  the family of all  $G$ -saturated ideals of  $End(G)$ .

The subset  $M$  of the algebra  $G$  is called the  $a(\sigma)$ -subset, if  $M$  is a union of subalgebras of the algebra  $G$ .

**Lemma 2.** *Let  $G$  be a universal algebra. The family  $Sub_{a(\sigma)}(G)$  of all  $a(\sigma)$ -subsets of the algebra  $G$  is a complete sublattice of the lattice  $Exp(G)$  of all subsets of  $G$ .*

The subset  $M$  of the algebra  $G$  is called the  $e(\sigma)$ -subset, if  $M = \cup\{\varphi(G) : \varphi \in H\}$  for some subset  $H \subset End(G)$ . The family  $Sub_{e(\sigma)}(G)$  of all  $e(\sigma)$ -subsets of the algebra  $G$  is a complete sublattice of the lattice  $Sub_{a(\sigma)}(G)$ .

**Theorem 1.** *Let  $G$  be a universal algebra. The following assertions are equivalent:*

1.  $G$  is a cyclic universal algebra.
2.  $\cup\{End(G)_{M_\gamma} : \gamma \in L\} = End(G)_{\cup\{M_\gamma : \gamma \in L\}}$  for any family of  $a(\sigma)$ -subsets  $\{M_\gamma : \gamma \in L\} \subset Sub_{a(\sigma)}(G)$ .

**Theorem 2** ([27], p. 79). *Let  $G$  be a free universal algebra. The following assertions are equivalent:*

1.  $G$  is a cyclic universal algebra.
2.  $\cup\{End(G)_{M_\gamma} : \gamma \in L\} = End(G)_{\cup\{M_\gamma : \gamma \in L\}}$  for any family of  $e(\sigma)$ -subsets  $\{M_\gamma : \gamma \in L\} \subset Sub_{e(\sigma)}(G)$ .

The following assertion was proved in [22, 23, 25, 27, 10].

**Theorem 3.** *Let  $G$  be a free universal algebra. Then  $Spec^s(S)$ ,  $Sub_{a(\sigma)}(G)$ ,  $Sub_{e(\sigma)}(G)$  are isomorphic complete lattices.*

G. Gratzner and E. T. Schmidt [17] proved that any complete lattice is isomorphic to the lattice of congruence of some universal algebra. The monoid  $End(G)$  of all endomorphisms of the universal algebra  $G$  is a semigroup with unity. In [20] A. I. Mal'cev describe the structure of a symmetrical groupoid (monoid of all transformations of a set). The following theorem is a generalization and conceptualization of the theorem from ([27], p. 98) and is proved in [10, 11].

**Theorem 4.** *For any monoid  $S$  there exist a signature  $\Omega$  and a universal algebra  $G_S$  of signature  $\Omega$  such that:*

1. *The semigroups  $S$  and  $End(G_S)$  are isomorphic.*
2.  *$G_S$  is a free cyclic universal algebra of signature  $\Omega$ .*

Let  $\alpha$  be a relation of equivalence on the universal algebra  $G$  of signature  $\Omega$  and  $p_\alpha : G \rightarrow G/\alpha$  be the natural projection. The binary relation  $\alpha$  is a congruence on  $G$  if and only if  $\alpha$  is an equivalence relation and on the set  $G/\alpha$  there exists a structure of a universal algebra of signature  $\Omega$  for which  $p_\alpha : G \rightarrow G/\alpha$  is a morphism. Let  $\alpha, \beta$  be two binary relations on the universal algebra  $G$ . The product of these relations  $\alpha\beta$  is defined as follows:  $x\alpha\beta y$  if and only if  $x\alpha z$  and  $z\beta y$  for some  $z \in G$ . If for any two congruences  $\alpha, \beta$  of the algebra  $G$  we will have  $\alpha\beta = \beta\alpha$ , then it is said that  $G$  is an algebra with permutable congruences. Algebra  $G$  is an algebra with correct (or regular) congruences if for any two different congruences  $\alpha, \beta$  there are no common equivalence classes: the sets  $\{y \in G : x\alpha y\}$  and  $\{y \in G : x\beta y\}$  are distinct for any point  $x \in G$ . These notions were introduced by G. Birkhoff in 1935 [3]. A. I. Mal'cev [19] showed the significance of these notions in algebra and topology, constructed an example of algebra with permutable congruences without correct congruences, constructed a class of biternary algebras in which congruences are permutable and correct. H. A. Thurston [21] in 1958 formulated the question: is algebra with permutable congruences an algebra with correct congruences? This problem was completely solved by I. Valuța in the paper [24]. The following results were obtained:

**Theorem 5.** *There is a universal algebra with six elements and two binary operations with correct congruences and without permutable congruences.*

**Theorem 6.** *If the universal algebra with correct congruences has at most five elements, then this algebra is with permutable congruences.*

These results have been published in papers [22, 23]. In the works [25, 26] Professor Ion Valuța obtained other profound results regarding the structure of some concrete lattice of morphisms. In particular, the ideals of the right are also described. In monograph [27] a general theory of the ideals of the abstract monoid  $End(G)$  for any free algebra  $G$  has been proposed. This research was continued in articles [10, 11, 12].

**History and Methodology of Science:** A special direction in the activity of professor Ion Valuța occupies the researches related to the history and methodology of science, especially of mathematics and informatics. Ion Valuța discussed at various scientific conferences the history of science development on Romanian lands [9, 29]. Science has emerged as a result of the formation of the relationship between human and nature. Viability, pragmatic spirit and reality, as well as the world of ideas and reasoning, represent

the effective and objective existence of science, a fact confirmed by the huge variety of its achievements in different areas of life and mind. These principles were applied by Professor Ion Valuță to the history of mathematical research in the Republic of Moldova, reflected in the chapters of the collective monograph [7]. In the study of the history and methodology of mathematics, Professor Ion Valuță was initially based on the periodization of the history of mathematics after A. N. Kolmogorov [18], who mentioned four periods of the history of mathematics. A. N. Kolmogorov's point of view was developed in [29] and then the history of the development of mathematics and computer science was divided into seven periods with six subperiods [8]. The notion of period reflects not so much the level obtained in a certain region, but the new ideas, methodological conceptions and mathematical tools developed at that time, regardless of the regions in which these results would have been obtained. An important role in the study of the history of mathematics, and generally of science, is played by the formation of scientific language and the formation of scientific concepts. We could say that the concepts demonstrate how natural processes and phenomena are formed and directed. The formation of terminology in the mother language is an important process for the culture and for the educational system of each nation. The name of the concepts, operations, symbols and other objects necessary to describe the scientific processes and conclusions, depends on the language in which the scientific work is presented. Professor Ion Valuță had an essential contribution to the formation of the mathematical language for the Romanian speakers in the area of the Republic of Moldova. He translated a series of textbooks from other languages into Romanian with Cyrillic spelling (which in 1938 was called the Moldavian language), contributed to the elaboration of the works [1, 32]. When elaborating these mathematical dictionaries, the authors were lead by the mathematical terminology that is used in Romania. For this reason, the "Russian-Moldavian Mathematical Dictionary" is a "Russian-Romanian Mathematical Dictionary", but written in Cyrillic characters. This fact did not go unnoticed. In the book ([13], p. 32) N. G. Corlateanu mentions "It should also be mentioned that in the field of language practice in the SSR Union, in general the principle of minimum differentiation between the languages of the Soviet people is applied. ... As for the new terms ... it is recommended to follow the principle of minimum differentiation. Usually, these new terms go through the Russian succession and there is no point in making an artificial difference from the way they appear in this language". So it was not easy to write Romanian in Cyrillic, because the people in the MSSR are a Soviet people and must respect the "internationalization of terminology" in terms of "the principle of minimum differentiation between the languages of Soviet peoples."

**Didactics and Education:** Professor Ion Valuța also carried out his activity in the study of the problems of the educational process: various problems of the organization of the educational process; principles and methods of teaching subjects. First, he studied the principles and methods of teaching mathematics in the training of teachers of mathematics, physics and computer science. He developed the course of history and methodology of mathematics [7]. Studying the role of mathematics and computer science in the training of highly qualified engineers, he developed the optional course "Basics of the theory of universal algebra" [28], "Elements of linear programming" [31] and monograph "Economic calculations based on optimal planning models" [5]. Together with G. D. Diligul, they studied the mathematical principles and bases in the process of training the secondary engineering staff with specialized secondary education. And, as a result, the textbook "Mathematics" [30] for engineering colleges was developed, which was selected by the Ministry of Higher and Specialized Education of the Soviet Union for all engineering colleges of the USSR.



In the position of the dean of the physical-mathematical faculty of Tiraspol University, Professor Ion Valuța raised the issue of improving the study process for students. First of all, the question was that the study process should correspond to the principles of continuity and inter-multi-disciplinarily. The formation of research skills is an inter-multi-disciplinary factor of continuity of studies. In this recent picture, professors Ion

Valuță and Sergiu Miron (born on the 3rd of September 1925) with a group of their former graduate students of the faculty from 1965.

**Management of Scientific Research:** For many years Professor Ion Valuță held various leadership positions and scientific projects, manifesting the qualities of a skilled organizer. He was the scientific leader of the doctoral students, training five doctors of science. He has been active in attesting scientific staff, since 1967 as an official referent or member of the scientific councils for defending doctoral thesis's in sciences, is an active member of the scientific seminar in algebra, mathematical logic and number theory. Since 1965 he has participated in all union scientific conferences of algebraists, at symposia on various fields of mathematics and its applications. Since 1992 he participates in the Conferences of the Romanian Society of Applied and Industrial Mathematics - CAIM. At his initiative were organized many scientific conferences in the Republic of Moldova, one of the last was CAIM-2019. He has published over a hundred scientific articles, over thirty books, five monographs and many publications of different character.

Recently, between October 27 and 28, 2020, was held the work of the International Symposium "Actual Problems of Mathematics and Informatics" dedicated to the 90th Birthday of Professor Ion Valuță. The above words confirms that Professor Ion Valuță is a multidimensional personality, full of patriotism, which completely corresponds to the characteristic of patriotism in the vision of the great Romanian writer Mihail Sadoveanu: "Patriotism does not mean hatred against other nations, but duty to our nation; it does not mean the claim that we are the most worthy people in the world, but the urge to become a worthy people. Honest work, clean living, love of fellow human beings, weaving the debts we have - that is, deeds - this means patriotism and not empty words.

HAPPY BIRTHDAY Professor Ion VALUȚĂ.

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